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# On the Possibility of Reducing the Objective Lens Diameter in the Optoelectronic System

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## Abstract

When optoelectronic systems (OES) are used for control purposes in mechanical engineering, there is a risk of objective cracking caused by collision with hard particles of the environment. The article shows that it is possible to reduce the objective lens diameter two times without loss of resolution, and thus to improve impact resistance by reducing the probability of collision and objective lens cracking.

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## 1. Introduction

When optoelectronic systems are designed to control the technological processes in mechanical engineering, it is necessary to take into account the risk of collision of the objective of an OES with hard particles of the environment [1-2]. This collision can cause not only scratches on the objective of an OES, but also its cracking and even splitting.

As mentioned in [3], optoelectronic systems may be applied for accurate measurements of rolled metal thickness. However, in this case the objective surface may also be damaged by the collision, for example, with the particles of the scale of moving flat rolled stock.

The above-mentioned reasons arouse interest in the search for the ways of reducing the diameter of OES objectives without a decrease in the resolution of the image of an object under observation.

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## 2. Estimation of the influence of the objective diameter on the objective diffraction limits

To solve this problem we will estimate the influence of the objective size on the resolution of an image generated by an OES. It is known that the propagation path of the image signal is determined by atmospheric turbulence, effects of optical diffraction, limited dimensions of OES receiver elements, final band pass of video amplifier, and other factors. As mentioned in [4], the path is equivalent to the filter of low space frequencies with an impulse characteristic (referred to as IC) that forms the output signal of an OES by convolution with the input signal

$$U_{out}(x, y) = \int_{G(\tau_x, \tau_y)} \int U_{in}(x + \tau_x, y + \tau_y) \cdot h_{tr}(\tau_x, \tau_y) \cdot d\tau_x \cdot d\tau_y. \quad (1)$$

Trying to estimate the output signal of an OES, we face two problems: the evaluation of the IC of the path  $h_{tr}(\tau_x, \tau_y)$  and determination of the integration domain  $G(\tau_x, \tau_y)$  of this IC.

To resolve these problems we can take into account [4, pp. 23, 295] that the IC  $h_{tr}(\tau_x, \tau_y)$  is a convolution of impulse characteristics for every factor, including diffraction. So, the corollary of the central limit theorem of the probability theory is usually used to estimate the IC  $h_{tr}(\tau_x, \tau_y)$  [5, p. 565]. This corollary tells that a convolution of several IC may be expressed as an IC of asymptotic Gaussian type

$$h_{tr.G}(\tau_x, \tau_y) \approx \frac{1}{\sqrt{2\pi} \cdot \sigma_{tr.x}} \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma_{tr.y}} \cdot \exp\left(-\frac{\tau_x^2}{2\sigma_{tr.x}^2}\right) \cdot \exp\left(-\frac{\tau_y^2}{2\sigma_{tr.y}^2}\right) \quad (2)$$

where  $\sigma_{tr.x} = \sqrt{\sum_i \sigma_{x,i}^2}$ ,  $\sigma_{tr.y} = \sqrt{\sum_i \sigma_{y,i}^2}$ ;  $\sigma_{x,i}, \sigma_{y,i}$  are mean square deviations (MSD) of each factor.

Considering the approximation (2), the calculation of the expression (1) can be simplified:

$$U_{out}(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U_{in}(x - \tau_x, y - \tau_y) \cdot h_{tr.G}(\tau_x, \tau_y) \cdot d\tau_x \cdot d\tau_y \quad (3)$$

Moreover, due to the separable variables in the IC (2) it is possible to confine the attention simply on one-dimensional analysis for every factor instead of finding resolution for two-dimensional problems (3). In particular, in the diffraction analysis the signal at the output of the objective is analyzed in the form of a convolution with the IC  $h_{dif}(x)$  of the objective, considering [4,6,7].

$$U_{out}(x) = \int_{-\infty}^{\infty} U_{in}(x - \tau) \cdot h_{dif}(\tau) \cdot d\tau. \quad (4)$$

The shape of an objective is usually round or rectangular. For a round objective, the IC has a relatively complex expression to be analyzed, while the IC  $h_{dif}(x)$  of a rectangular one has a more simple form

$$h_{dif}(x) = \frac{\sin^2(w_I \cdot x)}{(w_I \cdot x)^2} \quad (5)$$

It is known, that under certain conditions the IC of a round objective may be transformed to the IC of a rectangular one [4]. So, at first the MSD of the IC of a rectangular objective will be estimated below, and then we will consider this parameter for a round objective.

The IC  $h_{dif}(\bar{x})$  (5) as the function of dimensionless distance  $\bar{x} = x/w_I$  is shown as a heavy line on the Figure 1, with

$$w_I = \pi/\delta \quad (6)$$

where  $\delta$  is the first value which makes (5) equal to zero.

According to the central limit theorem of the probability theory, the function  $h_{dif}(x)$  (5) is asymptotically equivalent to the Gaussian function.

$$h_G(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma_G} \cdot \exp\left(-\frac{x^2}{2\sigma_G^2}\right), \quad (7)$$

$$\sigma_G = \sigma_{dif} \quad (8)$$

Considering [5], the value of the MSD  $\sigma_{dif}$  of the IC  $h_{dif}(x)$  in (5) is presented in the following form:

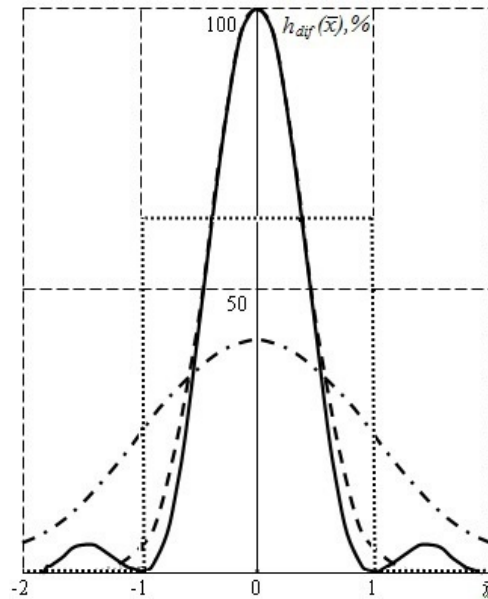


Figure 1. Approximation of the IC by the simple functions.

$$\sigma_{dif} = \frac{\sqrt{\int_{-\infty}^{\infty} h_{dif}(x) \cdot x^2 \cdot dx}}{\sqrt{\int_{-\infty}^{\infty} h_{dif}(x) \cdot dx}}. \quad (9)$$

Substitution of (5) into (9) gives the result

$$\sigma_{dif} = \infty. \quad (10)$$

It is known, that there are functions [8] with endless MSD (10); however, their convolution with other IC lead to Gaussian-like distributions with a limited MSD. Therefore, it is not always correct to use the estimation of the MSD of the IC (5) taken in the form (9) to estimate the MSD of the IC of the path (2), which is in contradiction with the central limit theorem of the probability theory [5].

To overcome the contradiction between the endless value of the MSD (9) of the IC (5) and the limited MSD of the equivalent IC (8), designers and theorists take the heuristic way. For instance, in Russia [7], as well as in other countries [6], designers often take the Gaussian form of the IC (7) and MSD as an approximation function (Figure 1, dash-dotted line)

$$\sigma_{dif} = \delta, \quad (11)$$

where  $\delta = \lambda_{av}/d$ ,  $\lambda_{av}$  is the average length of the signal wave,  $d$  is the objective diameter.

They also calculate convolution (4) using table data [9] or rectangular approximation of  $h_{dif}(x)$  (Figure 1, thin line). Designers abroad [10] choose MSD equal to  $\sigma_{dif} = \delta/\sqrt{3}$ , and Russian designers considering [9] take an even higher value

$$\sigma_{dif} = (2 \cdot \delta)/\sqrt{3}. \quad (12)$$

Both approximations of  $h_{dif}(x)$  with MSD (11) and (12) are obviously far from IC (5). This factor determines the relevance of the search for the solutions ensuring a more accurate estimation of the MSD of the IC (5).

To find such estimation of the MSD  $\sigma_{dif}$  of the IC (5), it is necessary to take into account the fact that the endless value of the MSD (10) arises from the large equivalent duration  $\Delta x_{aqu}$  of the IC (5) estimated by the following equation:

$$\Delta x_{aqu} = \frac{\int_{-\infty}^{\infty} h_{dif}(x) \cdot |x| \cdot dx}{\int_{-\infty}^{\infty} h_{dif}(x) \cdot dx}, \quad (13)$$

It is known that the large duration of the IC corresponds to the small width of the signal spectrum  $\Delta w_{aqu}$  as these parameters are inversely proportional [11]

$$\Delta w_{aqu} \approx \frac{2\pi}{\Delta x_{aqu}}. \quad (14)$$

The width of signal spectrum  $\Delta w_{aqu}$  is

$$\Delta w_{aqu} = \frac{\int_{-\infty}^{\infty} G_{dif}(w) \cdot |w| \cdot dw}{\int_{-\infty}^{\infty} G_{dif}(w) \cdot dw} \quad (15)$$

Where, considering [4],

$$G_{dif}(w) = \begin{cases} (1 - |w|/(2w_I)), & |w| < w_I, \\ 0, & |w| \geq w_I. \end{cases} \quad (16)$$

In (15) and (16),  $G_{dif}(w)$  is a spectrum function determined by the Fourier transformation of the IC  $h_{dif}(x)$  (5).

This function may be represented in the form

$$G_G(w) = \frac{1}{\sqrt{2\pi} \cdot w_{\sigma,G}} \cdot \exp\left(-\frac{w^2}{2w_{\sigma,G}^2}\right), \quad (17)$$

where  $w_{\sigma,G}$  is defined by the MSD of Gauss distribution (7) by the relation

$$w_{\sigma,G} = \frac{I}{\sigma_G}. \quad (18)$$

Now we determine the MSD of the spectrum (17)

$$w_{\sigma,dif} = \frac{\sqrt{\int_{-\infty}^{\infty} G_{dif}(w) \cdot w^2 \cdot dw}}{\sqrt{\int_{-\infty}^{\infty} G_{dif}(w) \cdot dw}} = w_I \cdot \sqrt{\frac{2}{3}}. \quad (19)$$

As it follows from the equation (19), the MSD of the spectrum (17) of the IC (4) is a limited value.

The essence of the proposed solution is immediate application of the central limit theorem of the probability theory to the spectrum (17) of the IC (4) with a limited MSD (19), but not to the IC (4) with an endless MSD (10). Moreover, if the IC, as the convolution of separate IC, must have an asymptotic Gaussian form with a corresponding MSD, then every convoluted IC has a Gaussian spectrum and a corresponding MSD. Thus, equating the MSD of the spectra (18) and (19) and taking into account the value of the MSD (8), we come to the MSD of the IC (7)

$$\sigma_{dif} = \frac{\sqrt{3/2}}{\pi} \cdot \delta. \quad (20)$$

The IC (5) with the Gaussian approximation (7) for a rectangular objective with a MSD (20) is shown in Figure 1 (dashed line). In the main domain of definition of the IC (5) and (7)

$$-\pi \cdot \sigma_{dif} \leq x \leq \pi \cdot \sigma_{dif} \quad (21)$$

the IC (5) and its approximation (7) coincide with a high degree of accuracy.

For the round form of objective [4], an approximation of the IC similar to (5) is effective

$$h_{dif}(x) \approx \frac{\sin^2(w_I^* \cdot x)}{(w_I^* \cdot x)^2}, \quad (22)$$

where  $w_I^* = \pi \cdot d^* / \lambda_{av}$ ,  $d^* \approx d/1.2$ . Comparing the expressions of the IC for a rectangular objective (5), (6), (11) with the equations (20) – (22) for a round objective, one can come to a conclusion that the diffraction of a round objective is equivalent to the diffraction of a rectangular objective if the MSD of diffraction increases approximately 1.2 times. Keeping it in mind, the expression (20) for the MSD of the IC for a round objective may be written in the form

$$\sigma_{dif}^* \approx 1.2 \cdot \frac{\sqrt{3/2}}{\pi} \cdot \delta. \quad (23)$$

The accuracy of the approximation of the obtained equations for rectangular (20) and round (23) objectives amounted to a few percent, which corresponds to the uncertainty in the estimation of other parameters of the optical path in the equation (2). The foregoing confirms the possibility of maintaining the required accuracy of the Gaussian approximation of the obtained expression of the IC (7).

Application of the results obtained in this work gives an opportunity to decrease the objective diameter at least two times without loss of the OES resolution. The obtained results are confirmed by experiments [1-3, 12-15].

### 3. Conclusion

In conclusion it is reasonable to note that some designers of OES choose a wrong shape of the approximation function, i.e. a rectangular shape, and incorrectly estimate the influence of the objective diameter on the diffraction value. But even in case when rational Gaussian approximation of the IC is chosen, designers in Russia as well as abroad mistake more than in two times estimating the MSD of the IC. It reduces the efficiency of OES, for instance, leads to an unjustified decrease of the impact resistance of OES and so on. The results represented in this paper give an opportunity to reduce the lens diameter more than four times, which will result in the improvement of the OES efficiency. The obtained results are confirmed by experiments.

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